EXERCISES FOR SECTION 1.3

LAB SERIES Lab 1.1

In Exercises 1–4, use a graphing utility to graph the function and visually estimate the limits.

1. $h(x) = x^2 - 5x$

(a) $\lim_{x \to 5} h(x)$

(b) $\lim_{x \to -1} h(x)$

2. $g(x) = \dfrac{12(\sqrt{x} - 3)}{x - 9}$

(a) $\lim_{x \to 4} g(x)$

(b) $\lim_{x \to 0} g(x)$

3. $f(x) = x \cos x$

(a) $\lim_{x \to 0} f(x)$

(b) $\lim_{x \to \pi/3} f(x)$

4. $f(t) = t|t - 4|$

(a) $\lim_{t \to 4} f(t)$

(b) $\lim_{t \to -1} f(t)$

In Exercises 5–28, find the limit.

5. $\lim_{x \to 4} x^2$

6. $\lim_{x \to -3} (3x + 2)$

7. $\lim_{x \to 0} (2x - 1)$

8. $\lim_{x \to 1} (-x^2 + 1)$

9. $\lim_{x \to 2} (-x^2 + x - 2)$

10. $\lim_{x \to 1} (3x^3 - 2x^2 + 4)$

11. $\lim_{x \to 3} \sqrt{x + 1}$

12. $\lim_{x \to 4} \sqrt[3]{x + 4}$

13. $\lim_{x \to -4} (x + 3)^2$

14. $\lim_{x \to 0} (2x - 1)^3$

15. $\lim_{x \to 2} \dfrac{1}{x}$

16. $\lim_{x \to -3} \dfrac{2}{x + 2}$

17. $\lim_{x \to -1} \dfrac{x^2 + 1}{x}$

18. $\lim_{x \to 3} \dfrac{\sqrt{x + 1}}{x - 4}$

19. $\lim_{x \to \pi/2} \sin x$

20. $\lim_{x \to \pi} \tan x$

21. $\lim_{x \to 1} \cos \pi x$

22. $\lim_{x \to 1} \sin \dfrac{\pi x}{2}$

23. $\lim_{x \to 0} \sec 2x$

24. $\lim_{x \to \pi} \cos 3x$

25. $\lim_{x \to 5\pi/6} \sin x$

26. $\lim_{x \to 5\pi/3} \cos x$

27. $\lim_{x \to 3} \tan\left(\dfrac{\pi x}{4}\right)$

28. $\lim_{x \to 7} \sec\left(\dfrac{\pi x}{6}\right)$

In Exercises 29–32, use the information to evaluate the limits.

29. $\lim_{x \to c} f(x) = 2$, $\lim_{x \to c} g(x) = 3$

(a) $\lim_{x \to c} [5g(x)]$

(b) $\lim_{x \to c} [f(x) + g(x)]$

(c) $\lim_{x \to c} [f(x)g(x)]$

(d) $\lim_{x \to c} \dfrac{f(x)}{g(x)}$

30. $\lim_{x \to c} f(x) = \frac{3}{2}$, $\lim_{x \to c} g(x) = \frac{1}{2}$

(a) $\lim_{x \to c} [4f(x)]$

(b) $\lim_{x \to c} [f(x) + g(x)]$

(c) $\lim_{x \to c} [f(x)g(x)]$

(d) $\lim_{x \to c} \dfrac{f(x)}{g(x)}$

31. $\lim_{x \to c} f(x) = 4$

(a) $\lim_{x \to c} [f(x)]^3$

(b) $\lim_{x \to c} \sqrt{f(x)}$

(c) $\lim_{x \to c} [3f(x)]$

(d) $\lim_{x \to c} [f(x)]^{3/2}$

32. $\lim_{x \to c} f(x) = 27$

(a) $\lim_{x \to c} \sqrt[3]{f(x)}$

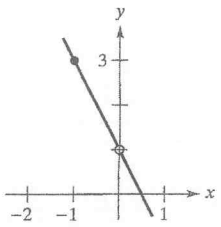
(b) $\lim_{x \to c} \dfrac{f(x)}{18}$

(c) $\lim_{x \to c} [f(x)]^2$

(d) $\lim_{x \to c} [f(x)]^{2/3}$

In Exercises 33–36, use the graph to determine the limit visually (if it exists). When possible, identify two functions that agree at all but one point.

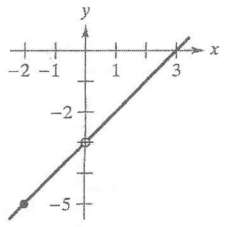
33. $g(x) = \dfrac{-2x^2 + x}{x}$



(a) $\lim_{x \to 0} g(x)$

(b) $\lim_{x \to -1} g(x)$

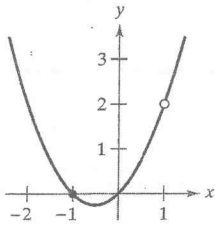
34. $h(x) = \dfrac{x^2 - 3x}{x}$



(a) $\lim_{x \to -2} h(x)$

(b) $\lim_{x \to 0} h(x)$

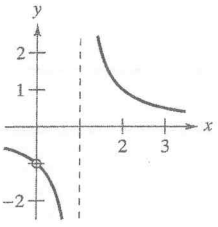
35. $g(x) = \dfrac{x^3 - x}{x - 1}$



(a) $\lim_{x \to 1} g(x)$

(b) $\lim_{x \to -1} g(x)$

36. $f(x) = \dfrac{x}{x^2 - x}$



(a) $\lim_{x \to 1} f(x)$

(b) $\lim_{x \to 0} f(x)$

In Exercises 37–40, find the limit of the function (if it exists). Identify two functions that agree at all but one point and use a graphing utility to graph the function.

37. $\lim_{x \to -1} \dfrac{x^2 - 1}{x + 1}$

38. $\lim_{x \to -1} \dfrac{2x^2 - x - 3}{x + 1}$

39. $\lim_{x \to -2} \dfrac{x^3 + 8}{x + 2}$

40. $\lim_{x \to -1} \dfrac{x^3 + 1}{x + 1}$

In Exercises 41–52, find the limit (if it exists).

41. $\lim_{x\to 5} \frac{x-5}{x^2-25}$

42. $\lim_{x\to 2} \frac{2-x}{x^2-4}$

43. $\lim_{x\to 1} \frac{x^2+x-2}{x^2-1}$

44. $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$

45. $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x}$

46. $\lim_{x\to 0} \frac{[1/(x+4)]-(1/4)}{x}$

47. $\lim_{x\to 0} \frac{[1/(2+x)]-(1/2)}{x}$

48. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$

49. $\lim_{\Delta x\to 0} \frac{2(x+\Delta x)-2x}{\Delta x}$

50. $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2-x^2}{\Delta x}$

51. $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2-2(x+\Delta x)+1-(x^2-2x+1)}{\Delta x}$

52. $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^3-x^3}{\Delta x}$

Graphical, Numerical, and Analytic Analysis **In Exercises 53–56, use a graphing utility to graph the function and estimate the limit. Use a table to reinforce your conclusion. Then find the limit by analytic methods.**

53. $\lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$

54. $\lim_{x\to 16} \frac{4-\sqrt{x}}{x-16}$

55. $\lim_{x\to 0} \frac{[1/(2+x)]-(1/2)}{x}$

56. $\lim_{x\to 2} \frac{x^5-32}{x-2}$

In Exercises 57–68, determine the limit of the trigonometric function (if it exists).

57. $\lim_{x\to 0} \frac{\sin x}{5x}$

58. $\lim_{x\to 0} \frac{3(1-\cos x)}{x}$

59. $\lim_{\theta\to 0} \frac{\sec\theta - 1}{\theta \sec\theta}$

60. $\lim_{\theta\to 0} \frac{\cos\theta \tan\theta}{\theta}$

61. $\lim_{x\to 0} \frac{\sin^2 x}{x}$

62. $\lim_{x\to 0} \frac{\tan^2 x}{x}$

63. $\lim_{h\to 0} \frac{(1-\cos h)^2}{h}$

64. $\lim_{\phi\to\pi} \phi \sec\phi$

65. $\lim_{x\to \pi/2} \frac{\cos x}{\cot x}$

66. $\lim_{x\to \pi/4} \frac{1-\tan x}{\sin x - \cos x}$

67. $\lim_{t\to 0} \frac{\sin^2 t}{t^2}$

[*Hint:* Find $\lim_{t\to 0}\left(\frac{\sin t}{t}\right)^2$.]

68. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$

[*Hint:* Find $\lim_{x\to 0}\left(\frac{2\sin 2x}{2x}\right)\left(\frac{3x}{3\sin 3x}\right)$.]

Graphical, Numerical, and Analytic Analysis **In Exercises 69–72, use a graphing utility to graph the function and estimate the limit. Use a table to reinforce your conclusion. Then find the limit by analytic methods.**

69. $\lim_{t\to 0} \frac{\sin 3t}{t}$

70. $\lim_{h\to 0} (1+\cos 2h)$

71. $\lim_{x\to 0} \frac{\sin x^2}{x}$

72. $\lim_{x\to 0} \frac{\sin x}{\sqrt[3]{x}}$

In Exercises 73–76, find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.

73. $f(x) = 2x + 3$

74. $f(x) = \sqrt{x}$

75. $f(x) = \frac{4}{x}$

76. $f(x) = x^2 - 4x$

In Exercises 77 and 78, use the Squeeze Theorem to find $\lim_{x\to c} f(x)$.

77. $c = 0$

$4 - x^2 \le f(x) \le 4 + x^2$

78. $c = a$

$b - |x-a| \le f(x) \le b + |x-a|$

In Exercises 79–84, use a graphing utility to graph the given function and the equations $y = |x|$ and $y = -|x|$ in the same viewing rectangle. Using the graphs to visually observe the Squeeze Theorem, find $\lim_{x\to 0} f(x)$.

79. $f(x) = x\cos x$

80. $f(x) = |x \sin x|$

81. $f(x) = |x| \sin x$

82. $f(x) = |x| \cos x$

83. $f(x) = x \sin\frac{1}{x}$

84. $h(x) = x \cos\frac{1}{x}$

85. ***Writing*** Use a graphing utility to graph

$$f(x) = x,\ g(x) = \sin x,\ \text{and } h(x) = \frac{\sin x}{x}$$

in the same viewing rectangle. Compare the magnitudes of $f(x)$ and $g(x)$ when x is "close to" 0. Use the comparison to write a short paragraph explaining why

$$\lim_{x\to 0} h(x) = 1.$$

86. ***Writing*** Use a graphing utility to graph

$$f(x) = x,\ g(x) = \sin^2 x,\ \text{and } h(x) = \frac{\sin^2 x}{x}$$

in the same viewing rectangle. Compare the magnitudes of $f(x)$ and $g(x)$ when x is "close to" 0. Use the comparison to write a short paragraph explaining why

$$\lim_{x\to 0} h(x) = 0.$$