Section 1.3

33 – 45, 57 – 65

33. Graph
    a. Limit at 0 is 1; the hole doesn't matter. The limit is the y-value
    b. Limit at -1 is 3 (y-value)
34. Graph
    a. Limit at -2 is -5
    b. Limit is -3 (y-values)
35. Graph
    a. Y-value is 2 (hole is unimportant)
    b. Y-value or limit is 0
36. Graph
    a. Limit does not exist because the graph does not come together at 0
    b. Limit is -1 (hole doesn't matter)

37. $$\lim_{x\to -1} \frac{x^2-1}{x+1}$$

$$= \lim_{x\to -1} \frac{(x+1)(x-1)}{x+1} \quad \text{// factor special case}$$

$$= \lim_{x\to -1} (x-1) \quad \text{// cancel repeat}$$

$$= -1-1$$

$$= -2$$

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38. $$\lim_{x\to -1} \frac{2x^2-x-3}{x+1}$$

$$= \lim_{x\to -1} \frac{(x+1)(2x-3)}{x+1} \quad \text{// factor leading coefficient not 1}$$

$$= \lim_{x\to -1} (2x-3) \quad \text{// cancel repeat}$$

$$= 2(-1)-3$$

$$= -2-3$$

$$= -5$$

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39. $\lim_{x \to -2} \dfrac{x^3 + 8}{x + 2}$

$= \lim_{x \to -2} \dfrac{(x+2)\left(x^2 - 2x + 4\right)}{x + 1}$ // factor cubes

$= \lim_{x \to -2} \left(x^2 - 2x + 4\right)$ // cancel repeat

$= (-2)^2 - 2(-2) + 4$

$= 4 + 4 + 4$

$= 12$

$= -5$

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40. $\lim_{x \to -1} \dfrac{x^3 + 1}{x + 1}$

$= \lim_{x \to -1} \dfrac{(x+1)\left(x^2 - x + 1\right)}{x + 1}$ // factor cubes

$= \lim_{x \to -1} \left(x^2 - x + 1\right)$ // cancel repeat

$= (-1)^2 - (-1) + 1$

$= 1 + 1 + 1$

$= 3$

$= -5$

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41. $\lim_{x \to 5} \dfrac{x - 5}{x^2 - 25}$

$= \lim_{x \to 5} \dfrac{x - 5}{(x+5)(x-5)}$ // factor special case

$= \lim_{x \to 5} \dfrac{1}{x + 5}$ // cancel repeat

$= \dfrac{1}{5 + 5}$

$= \dfrac{1}{10}$

42. $\lim_{x \to 2} \frac{2-x}{x^2-4}$

$= \lim_{x \to 2} \frac{2-x}{(x+2)(x-2)}$ // factor special case

$= \lim_{x \to 2} \frac{-(x-2)}{(x+2)(x-2)}$ // swap order when subtracting

$= \lim_{x \to 2} \frac{-1}{(x+2)}$ // cancel repeat

$= \frac{-1}{2+2}$

$= -\frac{1}{4}$

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43. $\lim_{x \to 1} \frac{x^2+x-2}{x^2-1}$

$= \lim_{x \to 1} \frac{(x+2)(x-1)}{(x+1)(x-1)}$ // factor

$= \lim_{x \to 1} \frac{(x+2)}{(x+1)}$ // cancel repeat

$= \frac{1+2}{1+1}$

$= \frac{3}{2}$

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44. $\lim_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$

$= \lim_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \dfrac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$ //rationalize

$= \lim_{x\to 0} \dfrac{\left(\sqrt{2+x}\right)^2+\sqrt{2}\sqrt{2+x}-\sqrt{2}\sqrt{2+x}-\left(\sqrt{2}\right)^2}{x\left(\sqrt{2+x}+\sqrt{2}\right)}$ //FOIL

$= \lim_{x\to 0} \dfrac{2+x-2}{x\left(\sqrt{2+x}+\sqrt{2}\right)}$ //cancel

$= \lim_{x\to 0} \dfrac{x}{x\left(\sqrt{2+x}+\sqrt{2}\right)}$

$= \lim_{x\to 0} \dfrac{1}{\sqrt{2+x}+\sqrt{2}}$

$= \dfrac{1}{\sqrt{2}+\sqrt{2}}$

$= \dfrac{1}{2\sqrt{2}}$

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45. $\lim_{x\to 0} \dfrac{\sqrt{3+x}-\sqrt{3}}{x}$

$= \lim_{x\to 0} \dfrac{\sqrt{3+x}-\sqrt{3}}{x} \cdot \dfrac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}}$ //rationalize

$= \lim_{x\to 0} \dfrac{\left(\sqrt{3+x}\right)^2+\sqrt{3}\sqrt{3+x}-\sqrt{3}\sqrt{3+x}-\left(\sqrt{3}\right)^2}{x\left(\sqrt{3+x}+\sqrt{3}\right)}$ //FOIL

$= \lim_{x\to 0} \dfrac{3+x-3}{x\left(\sqrt{3+x}+\sqrt{3}\right)}$ //cancel

$= \lim_{x\to 0} \dfrac{x}{x\left(\sqrt{3+x}+\sqrt{3}\right)}$

$= \lim_{x\to 0} \dfrac{1}{\sqrt{3+x}+\sqrt{3}}$

$= \dfrac{1}{\sqrt{3}+\sqrt{3}}$

$= \dfrac{1}{2\sqrt{3}}$

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57. $\lim_{x\to 0} \dfrac{\sin x}{5x}$

$= \lim_{x\to 0} \dfrac{1}{5} \cdot \dfrac{\sin x}{x}$

$= \dfrac{1}{5} \cdot 1$

$= \dfrac{1}{5}$

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58. $\lim_{x\to 0} \dfrac{3(1-\cos x)}{x}$

$= \lim_{x\to 0} 3 \cdot \dfrac{(1-\cos x)}{x}$

$= 3 \cdot 0$

$= 0$

59. $\lim_{\theta \to 0} \frac{\sec\theta - 1}{\theta \sec\theta}$

$= \lim_{\theta \to 0} \frac{(1/\cos\theta) - 1}{\theta(1/\cos\theta)}$ //$\sec\theta = \frac{1}{\cos\theta}$

$= \lim_{\theta \to 0} \frac{(1/\cos\theta) - 1}{\theta(1/\cos\theta)} \cdot \frac{\cos\theta}{\cos\theta}$ // multiply same thing on top/bottom to cancel complex fractions of 1/cos

$= \lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta}$

$= 0$ // from formula

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60. $\lim_{\theta \to 0} \frac{\cos\theta \tan\theta}{\theta}$

$= \lim_{\theta \to 0} \frac{\cos\theta \cdot \frac{\sin\theta}{\cos\theta}}{\theta}$ //$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$ //cancel cos in numerator

$= 1$ // from formula

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61. $\lim_{x \to 0} \frac{\sin^2 x}{x}$

$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1}$ //$\sin^2 x = \sin x \cdot \sin x$

$= 1 \cdot \frac{\sin 0}{1}$ // formula

$= 1 \cdot 1$

$= 1$

62.

$$\lim_{x\to 0}\frac{\tan^2 x}{x}$$

$$= \lim_{x\to 0}\frac{\tan x}{x}\cdot\frac{\tan x}{1} \quad //\tan^2 x = \tan x\cdot\tan x$$

$$= \lim_{x\to 0}\frac{\sin x}{x\cos x}\cdot\frac{\tan x}{1} \quad //\tan x = \frac{\sin x}{\cos x}$$

$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\tan x}{\cos x} \quad //\text{ doesn't matter order when multiplying}$$

$$= 1\cdot\frac{\tan 0}{\cos 0}$$

$$= \frac{0}{1} \quad //\text{ calculator}$$

$$= 0$$

63.

$$\lim_{h\to 0}\frac{(1-\cos h)^2}{h}$$

$$= \lim_{h\to 0}\frac{(1-\cos h)}{h}\cdot\frac{1-\cos h}{1} \quad //(1-\cos h)^2 = (1-\cos h)(1-\cos h)$$

$$= 0\cdot\frac{1-\cos 0}{1} \quad //\text{ formula}$$

$$= 0\cdot\frac{1-1}{1}$$

$$= 0$$

64.

$$\lim_{\phi\to \pi}\phi\sec\phi$$

$$= \lim_{\phi\to \pi}\phi\cdot\frac{1}{\cos\phi}$$

$$= \pi\cdot\frac{1}{\cos\pi}$$

$$= \pi\cdot\frac{1}{-1}$$

$$= -\pi$$

65.

$$\lim_{x \to \pi/2} \frac{\cos x}{\cot x}$$

$$= \lim_{x \to \pi/2} \frac{\cos x}{1/\tan x} \quad //\cot x = \frac{1}{\tan x}$$

$$= \lim_{x \to \pi/2} \tan x \cos x \quad //\text{reciprocal of } 1/\tan \text{ is } \tan$$

$$= \lim_{x \to \pi/2} \frac{\sin x}{\cos x} \cdot \cos x \quad //\tan x = \frac{\sin x}{\cos x}$$

$$= \lim_{x \to \pi/2} \sin x \quad //\textit{cancel}$$

$$= \sin \frac{\pi}{2}$$

$$= 1 \quad //\text{calculator}$$