Section 2.3

#13 – 15, 27 – 40, 42, 73 – 82

13.

$$f(x) = \frac{3x-2}{2x-3}$$

$$u = 3x-2$$

$$u' = 3$$

$$v = 2x-3$$

$$v' = 2$$

$$\frac{u'v - v'u}{v^2}$$

$$\frac{3(2x-3) - 2(3x-2)}{(2x-3)^2}$$

$$= \frac{6x-9-6x+4}{(2x-3)^2}$$

$$= \frac{-5}{(2x-3)^2}$$

14.

$$f(x) = \frac{x^3 + 3x + 2}{x^2 - 1}$$

$$u = x^3 + 3x + 2$$

$$u' = 3x^2 + 3$$

$$v = x^2 - 1$$

$$v' = 2x$$

$$\frac{u'v - v'u}{v^2}$$

$$\frac{\left(3x^2 + 3\right)\left(x^2 - 1\right) - 2x\left(x^3 + 3x + 2\right)}{\left(x^2 - 1\right)^2}$$

$$= \frac{3x^4 - 3x^2 + 3x^2 - 3 - 2x^4 - 6x^2 - 4x}{\left(x^2 - 1\right)^2}$$

$$= \frac{x^4 - 6x^2 - 4x - 3}{\left(x^2 - 1\right)^2}$$

15.

$$f(x) = \frac{3 - 2x - x^2}{x^2 - 1}$$

$$u = 3 - 2x - x^2$$

$$u' = -2 - 2x$$

$$v = x^2 - 1$$

$$v' = 2x$$

$$\frac{u'v - v'u}{v^2}$$

$$\frac{(-2 - 2x)\left(x^2 - 1\right) - 2x\left(3 - 2x - x^2\right)}{\left(x^2 - 1\right)^2}$$

$$= \frac{-2x^2 + 2 - 2x^3 + 2x - 6x + 4x^2 + 2x^3}{\left(x^2 - 1\right)^2}$$

$$= \frac{2x^2 - 4x + 2}{\left(x^2 - 1\right)^2}$$

27.

$$f(t) = t^2 \sin t$$

$$u = t^2$$

$$u' = 2t$$

$$v = \sin t$$

$$v' = \cos t$$

$$u'v + v'u$$

$$2t(\sin t) + (\cos t)t^2$$

$$2t \sin t + t^2 \cos t$$

28.

$f(\theta) = (\theta + 1)\cos\theta$

$u = (\theta + 1)$

$u' = 1$

$v = \cos\theta$

$v' = -\sin\theta$

$u'v + v'u$

$1(\cos\theta) + (-\sin\theta)(\theta + 1)$

$\cos\theta - \theta\sin\theta - \sin\theta$

29.

$f(t) = \frac{\cos t}{t}$

$u = \cos t$

$u' = -\sin t$

$v = t$

$v' = 1$

$\frac{u'v - v'u}{v^2}$

$\frac{(-\sin t)(t) - 1(\cos t)}{t^2}$

$= \frac{-t\sin t - \cos t}{t^2}$

30.

$$f(x) = \frac{\sin x}{x}$$

$u = \sin x$

$u' = \cos x$

$v = x$

$v' = 1$

$$\frac{u'v - v'u}{v^2}$$

$$\frac{(\cos x)(x) - 1(\sin x)}{x^2}$$

$$\frac{x\cos x - \sin x}{x^2}$$

31.

$f(x) = -x + \tan x$

$f'(x) = -1 + \sec^2 x$

32.

$y = x + \cot x$

$y' = 1 - \csc^2 x$

33.

$$g(t) = \sqrt{t} + 4\sec t$$

$$g(t) = t^{\frac{1}{2}} + 4\sec t$$

$$g'(t) = \frac{1}{2}t^{\frac{1}{2}-1} + 4(\sec x \tan x)$$

$$g'(t) = \frac{1}{2}t^{\frac{-1}{2}} + 4\sec x \tan x$$

$$g'(t) = \frac{1}{2t^{\frac{1}{2}}} + 4\sec x \tan x$$

$$g'(t) = \frac{1}{2\sqrt{t}} + 4\sec x \tan x$$

34.

$$h(s) = \frac{1}{s} - 10\csc s$$

$$h(s) = s^{-1} - 10\csc s$$

$$h'(s) = -s^{-2} - 10(-\csc s \cot s)$$

$$h'(s) = \frac{-1}{s^2} + 10\csc s \cot s$$

37.

$$y = -\csc x - \sin x$$

$$y' = -(-\csc x \cot x) - (\cos x)$$

$$y' = \csc x \cot x - \cos x$$

38.

$y = x\sin x + \cos x$

$u = x$

$u' = 1$

$v = \sin x$

$v' = \cos x$

$y' = 1(\sin x) + (\cos x)(x) + (-\sin x)$

$y' = \sin x + x\cos x - \sin x$

39.

$y = x^2 \sin x + 2x\cos x$

$u = x^2$

$u' = 2x$

$v = \sin x$

$v' = \cos x$

$u = 2x$

$u' = 2$

$v = \cos x$

$v' = -\sin x$

$y' = 2x(\sin x) + (\cos x)\left(x^2\right) + 2x(\cos x) + (-\sin x)(2x)$

$y' = 2x\sin x + x^2\cos x + 2x\cos x - 2x\sin x$

40.

$f(x) = \sin x \cos x$

$u = \sin x$

$u' = \cos x$

$v = \cos x$

$v' = -\sin x$

$f'(x) = u'v + v'u$

$f'(x) = \cos x(\cos x) + (-\sin x)(\sin x)$

$f'(x) = \cos^2 x - \sin^2 x$

42.

$h(\theta) = 5\sec\theta + \tan\theta$

$h'(\theta) = 5\sec x \tan x + \sec^2 x$

73.

$f(x) = 4x^{\frac{3}{2}}$

$f'(x) = 4\left(\frac{3}{2}\right)x^{\frac{3}{2}-1}$

$f'(x) = 6x^{\frac{1}{2}}$

$f''(x) = 6\left(\frac{1}{2}\right)x^{\frac{1}{2}-1}$

$f''(x) = 3x^{\frac{-1}{2}}$

$f''(x) = \frac{3}{x^{\frac{1}{2}}}$

$f''(x) = \frac{3}{\sqrt{x}}$

74.

$$f(x) = \frac{x^2 + 2x - 1}{x}$$

$$u = x^2 + 2x - 1; \quad u' = 2x + 2; \quad v = x; \quad v' = 1$$

$$f'(x) = \frac{(2x + 2)(x) - 1\left(x^2 + 2x - 1\right)}{x^2}$$

$$f'(x) = \frac{2x^2 + 2x - x^2 - 2x + 1}{x^2}$$

$$f'(x) = \frac{x^2 + 1}{x^2}$$

$$u = x^2 + 1; \quad u' = 2x; \quad v = x^2; \quad v' = 2x$$

$$f''(x) = \frac{(2x)\left(x^2\right) - (2x)\left(x^2 + 1\right)}{\left(x^2\right)^2}$$

$$f''(x) = \frac{2x^3 - 2x^3 - 2x}{x^4}$$

$$f''(x) = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

75.

$$f(x) = \frac{x}{x-1}$$

$$u = x; \quad u' = 1; \quad v = x-1; \quad v' = 1$$

$$f'(x) = \frac{1(x-1) - 1(x)}{(x-1)^2}$$

$$f'(x) = \frac{x-1-x}{(x-1)^2}$$

$$f'(x) = \frac{-1}{(x-1)^2}$$

$$u = -1; \quad u' = 0;$$

$$v = (x-1)^2 = (x-1)(x-1) = x^2 - x - x + 1 = x^2 - 2x + 1$$

$$v' = 2x - 2$$

$$f''(x) = \frac{0\left(x^2 - 2x + 1\right) - (2x-2)(-1)}{\left((x-1)^2\right)^2}$$

$$f'' = \frac{2x-2}{(x-1)^4} = \frac{2(x-1)}{(x-1)^4} = \frac{2}{(x-1)^3}$$

76.

$$f(x) = x + \frac{32}{x^2}$$

$$f(x) = x + 32x^{-2}$$

$$f'(x) = 1 - 32(2)x^{-2-1}$$

$$f'(x) = 1 - 64x^{-3}$$

$$f''(x) = 0 - 64(-3)x^{-4}$$

$$f''(x) = 192x^{-4}$$

$$f''(x) = \frac{192}{x^4}$$

77.

$f(x) = 3\sin x$

$f'(x) = 3(\cos x) = 3\cos x$

$f''(x) = 3(-\sin x)$

$f'''(x) = -3\sin x$

78.

$f(x) = \sec x$

$f'(x) = \sec x \tan x$

$u = \sec x; \quad u' = \sec x \tan x; \quad v = \tan x; \quad v' = \sec^2 x$

$f''(x) = (\sec x \tan x)(\tan x) + \left(\sec^2 x\right)(\sec x)$

$f'''(x) = \sec x \tan^2 x + \sec^3 x = \sec x\left(\tan^2 x + \sec^2 x\right)$

79.

$f'(x) = x^2$

$f'''(x) = 2x$

80.

$f''(x) = 2 - \frac{2}{x} = 2 - 2x^{-1}$

$f'''(x) = 2 - 2(-1)x^{-2-1}$

$f''''(x) = 2 + 2x^{-3}$

$f''''(x) = 2 + \frac{2}{x^3}$

81.

$$f''''(x) = 2\sqrt{x} = 2x^{\frac{1}{2}}$$

$$f^{(4)} = 2\left(\frac{1}{2}\right)x^{\frac{1}{2}-1}$$

$$f^{(4)} = x^{\frac{-1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$$

82.

$$f^{(4)}(x) = 2x + 1$$

$$f^{(5)}(x) = 2$$

$$f^{(6)}(x) = 0$$